SPATIAL MODELS FOR DISTANCE SAMPLING DATA:
RECENT DEVELOPMENTS AND FUTURE DIRECTIONS

APPENDIX D: DETAILS OF THE TWEEDIE DISTRIBUTION

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1. INTRODUCTION

This appendix gives a brief mathematical explanation of the Tweedie distribution.

2. THE TWEEDIE DISTRIBUTION

The Tweedie distribution has three parameters: a mean (μ), dispersion (ϕ) and a third, power parameter (p), which leads to additional flexibility. The Tweedie distribution is characterised by the mean-variance relationship $\text{var}(Y) = \phi\mu^p$. Setting $p = 1$ gives a quasi-Poisson distribution and $p = 2$ gives a gamma distribution. Tweedie random variables are a sum of M gamma variables where M is Poisson distributed (Jørgensen, 1987).

The Tweedie distribution has the following PDF (for $1 < p < 2$)):

$$f(y;\mu,\phi,p) = a(y;\phi)\exp\left[\frac{1}{\phi}\left\{y\frac{\mu^{1-p}}{1-p} - \frac{\mu^{2-p}}{2-p}\right\}\right],$$

where

$$a(y;\phi) = \frac{1}{y}\sum_{j=1}^{\infty}\frac{y^{-j\alpha}(p-1)^{\alpha j}}{\phi^{j(1-\alpha)}(2-p)^j j!\Gamma(-j\alpha)}, \qquad \alpha = \frac{2-p}{1-p}$$

Further technical information can be found in Jørgensen (1987); Dunn and Smyth (2005) and practical applications can be found in Candy (2004); Shono (2008); Peel et al. (2012).

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